Коши - Риман

$v = x^3 - 3xy^2$, $w(i) = 1$ $\quad w(0+1\cdot i) = 1 + 0\cdot i$

$x = 0 \quad u = 1$

$y = 1 \quad v = 0$

$v'_x = 3x^2 - 3y^2 \qquad v'_y = -6xy$

$v''_{xx} = 6x \qquad v''_{yy} = -6x$

$v''_{xx} + v''_{yy} = 0$

$6x - 6x = 0 \Rightarrow$ ~~аналитична~~ гармонична

$u'_x = -6xy$

$u = \int 6xy\,dx = \frac{6y}{2}\int x\,dx^2 = 3y\int dx^2 = -3yx^2 + C(y)$

$(-3yx^2 + C(y))'_y = -3x^2 + 3y^2$

$-3x^2 + C'(y) = -3x^2 + 3y^2$

$C'(y) = +3y^2$

$C(x) = \int +3y^2\,dy = y^3 + A$

$c(y) = y^3 + A$

$\Rightarrow u = -3yx^2 + y^3 + A \quad | \; u = 1, x = 0, y = 0$

$1 = 3\cdot 0^2 \cdot 1 + 1^3 + A$

$1 - 1 = A$

$A = 0$

$\frac{y^3 i}{-1} = -1 \cdot y^3 \cdot i = i^2 \cdot y^3 \cdot i = y^3 i^3$

$\Leftrightarrow u = -3yx^2 + y^3 + 0$

$\Rightarrow w = -3yx^2 + y^3 + i(x^3 - 3xy^2) = -3yx^2 + y^3 + x^3 i - 3xy^2 i =$

$= (x^3 i + 3yx^2 - 3xy^2 i + y^3)\frac{i}{i} = i\left(\frac{x^3 i}{i} + \frac{3i^2 yx^2}{i} - \frac{3xy^2 i}{i} + \frac{y^3 i}{i\cdot i}\right) =$

$= i\left(x^3 + 3x^2 yi + 3xy^2 i^2 + \frac{y^3 \cdot i}{-1}\right) =$

$= i(x^3 + 3x^2 yi + 3xy^2 i^2 + y^3 \cdot i^3) = iz^3$