$$I=\int_{-\pi}^{\pi}\frac{7+x-x^3}{2-|\sin 7x|}dx.$$

Решение:

$y=\sin 7x$ е периодична с период $\ell=\dfrac{2\pi}{7}$.

Т1 Ака $y=f(x)$ е периодична с период ℓ и непрекъсната, $f:\mathbb{R}\to\mathbb{R}$, то

$$\int_{a+\ell}^{b+\ell}f(x)dx=\int_a^b f(x)dx.$$

Доказателство :

Нека $x=t+\ell\Rightarrow dx=dt\Rightarrow \displaystyle\int_{a+\ell}^{b+\ell}f(x)dx=\int_a^b f(t+\ell)dt=\int_a^b f(t)dt=\int_a^b f(x)dx.$

$$I=\int_{-\pi}^{\pi}\frac{7+x-x^3}{2-|\sin 7x|}dx=\int_{-\frac{\pi}{7}-6\frac{\pi}{7}}^{\frac{\pi}{7}+6\frac{\pi}{7}}\frac{7+x-x^3}{2-|\sin 7x|}dx=\int_{-\frac{\pi}{7}}^{\frac{\pi}{7}}\frac{7+x-x^3}{2-|\sin 7x|}dx\Rightarrow$$

$$I=\int_{-\frac{\pi}{7}}^{0}\frac{7+x-x^3}{2+\sin 7x}dx+\int_0^{\frac{\pi}{7}}\frac{7+x-x^3}{2-\sin 7x}dx\Rightarrow$$

$$I=7\int_{-\frac{\pi}{7}}^{0}\frac{dx}{2+\sin 7x}+\int_{-\frac{\pi}{7}}^{0}\frac{xdx}{2+\sin 7x}-\int_{-\frac{\pi}{7}}^{0}\frac{x^3dx}{2+\sin 7x}+7\int_0^{\frac{\pi}{7}}\frac{dx}{2-\sin 7x}+\int_0^{\frac{\pi}{7}}\frac{xdx}{2-\sin 7x}-\int_0^{\frac{\pi}{7}}\frac{x^3dx}{2-\sin 7x}.$$

Ще докажа, че : а) $\displaystyle\int_0^{\frac{\pi}{7}}\frac{dx}{2-\sin 7x}=\int_{-\frac{\pi}{7}}^{0}\frac{dx}{2+\sin 7x}$; б) $\displaystyle\int_0^{\frac{\pi}{7}}\frac{xdx}{2-\sin 7x}=-\int_{-\frac{\pi}{7}}^{0}\frac{xdx}{2+\sin 7x}$;

в) $\displaystyle\int_{-\frac{\pi}{7}}^{0}\frac{x^3dx}{2+\sin 7x}=-\int_0^{\frac{\pi}{7}}\frac{x^3dx}{2-\sin 7x}$

Доказателство :

б) $\displaystyle\int_0^{\frac{\pi}{7}}\frac{xdx}{2-\sin 7x}=-\int_{-\frac{\pi}{7}}^{0}\frac{xdx}{2+\sin 7x}$ (а) и в) се получават аналогично)