

Нека $x = -t \Rightarrow dx = -dt \Rightarrow \int_0^{\frac{\pi}{7}} \frac{x dx}{2 - \sin 7x} = \int_0^{-\frac{\pi}{7}} \frac{(-t)(-dt)}{2 - \sin(-7t)} = - \int_{-\frac{\pi}{7}}^0 \frac{t dt}{2 + \sin 7t} = - \int_{-\frac{\pi}{7}}^0 \frac{x dx}{2 + \sin 7x}.$

Така $I = 2 \cdot 7 \int_0^{\frac{\pi}{7}} \frac{dx}{2 - \sin 7x} = 14 \int_0^{\frac{\pi}{7}} \frac{dx}{2 - \sin 7x} = 14 I_1$, където $I_1 = \int_0^{\frac{\pi}{7}} \frac{dx}{2 - \sin 7x}.$

$$I_1 = \int_0^{\frac{\pi}{7}} \frac{dx}{2 - \sin 7x} = \frac{x}{2 - \sin 7x} \Big|_0^{\frac{\pi}{7}} - \int_0^{\frac{\pi}{7}} \frac{7x \cos 7x dx}{(2 - \sin 7x)^2} = \frac{\pi}{14} - 7 \cdot \frac{\pi}{14} \int_0^{\frac{\pi}{7}} \frac{\cos x dx}{(2 - \sin 7x)^2}.$$

$$\left(\int_0^{\frac{\pi}{7}} x f(x) dx = \frac{\pi}{14} \int_0^{\frac{\pi}{7}} f(x) dx, \text{ ако } f(x) \text{ е непрекъсната} \right) \Rightarrow$$

$$I_1 = \frac{\pi}{14} + \frac{\pi}{2} \int_0^{\frac{\pi}{7}} \frac{d(2 - \sin 7x)}{(2 - \sin 7x)^2} = \frac{\pi}{14} - \frac{\pi}{2} (2 - \sin 7x) \Big|_0^{\frac{\pi}{7}} = \frac{\pi}{14} \Rightarrow I = \pi.$$